



Subject: Grade 10 Mathematics

Topic: Mathematics: Term 1 - Mini-Exam

Total: 105 Marks

1. C: $v = \frac{fu}{u - f}$

4 marks

Explanation:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

Isolate the term with v :

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} \quad \text{Add } -\frac{1}{u} \text{ both sides}$$

Find the LCD on the RHS: LCD is fu

$$\frac{1}{v} = \frac{1 \times u - 1 \times f}{fu}$$

$$\frac{1}{v} = \frac{u - f}{fu}$$

Swop both sides when you have 1 term on each side:

$$v = \frac{fu}{u - f}$$



2. B: $-\frac{31}{\sqrt{74}}$

4 marks

Explanation:

$\tan \theta = -\frac{7}{5}$ and $\tan \theta = \frac{y}{x}$, the coordinates of A is $(5; -7)$
 $\therefore x = 5$ and $y = -7$

Use Pythagoras to determine r (or the hypotenuse):

$$OA^2 = 5^2 + (-7)^2$$

$$OA^2 = 74$$

$$OA = \pm\sqrt{74}$$

But the hypotenuse is always positive, therefore $OA = \sqrt{74}$.

Use the triangle to determine adjacent and opposite with respect to θ :

$$\therefore x = \text{adj} = 5 \text{ and } y = \text{opp} = -7$$

Calculate: $3 \sin \theta - 2 \cos \theta$

$$= 3 \times \frac{\text{opp}}{\text{hyp}} - 2 \times \frac{\text{adj}}{\text{hyp}}$$

$$= 3 \times \frac{-7}{\sqrt{74}} - 2 \times \frac{5}{\sqrt{74}}$$

$$= \frac{-21}{\sqrt{74}} - \frac{10}{\sqrt{74}}$$

$$= -\frac{31}{\sqrt{74}}$$

Explanation:Given: $13 \cos \theta = -12$

$$\therefore \cos \theta = \frac{-12}{13}.$$

The ratio is $\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r}$ therefore $x = -12$ and $r = 13$.

The length of $OP = 13$.

- Coordinates of P are $(-12; y)$ in Quadrant 3.

Calculate the y -coordinate by using Pythagoras:

$$\begin{aligned}x^2 + y^2 &= r^2 \\(-12)^2 + y^2 &= 13^2 \\y^2 &= 169 - 144 \\y^2 &= 25 \\y &= \pm 5\end{aligned}$$

But y is **negative** in Quadrant 3, therefore coordinates of P are $(-12; -5)$

- $\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{y}{x}$

$$\therefore \tan \theta = \frac{-5}{-12} = \frac{5}{12}$$

4. D: $\frac{16}{x^{16}}$

4 marks

Explanation:

$$\left(\frac{x^4 + x^4}{x^4 \times x^4}\right)^4$$

Apply Exponent Law 1 in the denominator

$$= \left(\frac{2x^4}{x^8}\right)^4$$

$$\frac{x^4}{x^8} = x^{4-8} = x^{-4}$$

$$= (2x^{-4})^4$$

$$= \left(\frac{2}{x^4}\right)^4$$

$$x^{-4} = \frac{1}{x^4}$$

$$= \frac{2^4}{x^{16}}$$

Apply Exponent Law 5

$$= \frac{16}{x^{16}}$$

5. false

2 marks

Explanation:

$$1 - \frac{1}{64a^3}$$

$$\text{Answer: } \left(1 - \frac{1}{4a}\right)\left(1 + \frac{1}{4a} + \frac{1}{16a^2}\right)$$

6. $-1 \mid x = -1$

3 marks

Explanation:

$$2 \cdot \left(\frac{1}{2}\right)^{-x} - 5^{-2} \cdot 5^2 = 0$$

$$2 \cdot \left(\frac{1}{2}\right)^{-x} = 5^{-2} \cdot 5^2$$

Add $5^{-2} \cdot 5^2$ on both sides

$$2 \cdot (2^{-1})^{-x} = 5^0$$

Definition 3

$$2^1 \cdot 2^x = 1$$

Simplify, law 3

$$2^{1+x} = 2^0$$

Law 1

$$1 + x = 0$$

Equate exponents when bases are the same

$$x = -1$$

7. 3/4

3 marks

Explanation:

$$\begin{aligned} & \sin 45^\circ \sec 45^\circ + \cos^2 30^\circ - \frac{\tan 30^\circ}{\cot 60^\circ} \\ &= \frac{\sqrt{2}}{2} \cdot \frac{2}{\sqrt{2}} + \left(\frac{\sqrt{3}}{2}\right)^2 - \tan 30^\circ \times \tan 60^\circ \\ &= 1 + \frac{3}{4} - \frac{\sqrt{3}}{3} \cdot \sqrt{3} \\ &= 1 + \frac{3}{4} - 1 \\ &= \frac{3}{4} \end{aligned}$$

8. $\frac{1}{64}x^{12}$

3 marks

Explanation:

$$\begin{aligned} & [(-2x^2)^3]^{-2} \\ &= (-8x^6)^{-2} \quad \text{Apply the exponent law: } (ab)^m = a^m b^m \\ &= \frac{1}{(-8x^6)^2} \\ &= \frac{1}{64x^{12}} \end{aligned}$$

9. $x > -6 \mid x \in (-6; \infty)$

4 marks

Explanation:

$$\begin{aligned} 6x - 3(x + 4) &< 5x && \text{Multiply brackets} \\ 6x - 3x - 12 &< 5x \\ 3x - 5x &< 12 && \text{Arrange like terms on the same side} \\ -2x &< 12 \end{aligned}$$

Change the inequality sign when dividing by negative number:
 $x > -6$

The answer is $x > -6$.

In interval notation, it will be $x \in (-6; \infty)$.

Explanation:

Let x be the number of lollipops and the cost is R4,00 per lollipop.

Then $10 - x$ is the number of liquorice at a cost of R4,50 per liquorice.

Set up the equation:

$$x \times 4,00 + (10 - x) \times 4,50 = 43,50$$

Remove the brackets by multiplication:

$$\begin{aligned} 4x + 45 - 4,5x &= 43,50 \\ -0,5x &= -1,5 \\ \therefore x &= 3 \end{aligned}$$

Peter has bought $10 - x$ liquorice at R4,50 per liquorice.

$x = 3$ therefore he buys $10 - 3 = 7$ liquorice.

The cost of the liquorice is R31,50.

Explanation:

$$2x = 5y + 1 \quad \text{Equation 1}$$

$$24 - 7x = 3y \quad \text{Equation 2}$$

Change the subject in equation 1 to x by dividing by 2:

$$x = \frac{5y}{2} + \frac{1}{2} \quad \text{Equation 3}$$

Substitute equation 3 in equation 2:

$$24 - 7\left(\frac{5y}{2} + \frac{1}{2}\right) = 3y$$

$$24 - \frac{35y}{2} - \frac{7}{2} = 3y \quad \text{Multiply 7 into the brackets}$$

Multiply each term by 2:

$$48 - 35y - 7 = 6y$$

$$41 - 35y = 6y \quad \text{Simplify}$$

$$-41y = -41 \quad \text{Divide by -41}$$

$$y = 1$$

Substitute $y = 1$ in equation 3:

$$x = \frac{5(1)}{2} + \frac{1}{2}$$

$$x = 3$$

$\therefore$ The solution is (3; 1)

12. true

2 marks

Explanation:

$$\frac{3^{n+4} - 6 \cdot 3^{n+1}}{7 \cdot 3^{n+2}}$$

$$= \frac{3^n \cdot 3^4 - 6 \cdot 3^n \cdot 3^1}{7 \cdot 3^n \cdot 3^2} \quad \text{Write each exponent with its own base}$$

$$= \frac{3^n(3^4 - 6 \cdot 3^1)}{7 \cdot 3^n \cdot 3^2} \quad \text{Take out } 3^n \text{ as a common factor}$$

$$= \frac{3^4 - 6 \cdot 3^1}{7 \cdot 3^2} \quad \text{Divide numerator and denominator by } 3^n$$

$$= \frac{81 - 18}{63} \quad \text{Simplify all the powers}$$

$$= \frac{63}{63} = 1$$

13. D: $\frac{-(x+6)}{x}$

4 marks

Explanation:

$$\frac{x}{4-x} - \frac{2}{x} \times \frac{x-12}{x-4}$$

$$= \frac{x}{4-x} - \frac{2(x-12)}{x(x-4)}$$

Change the sign of $(x-4)$ to: $x-4 = -(4-x)$

$$= \frac{x}{4-x} - \frac{2(x-12)}{-x(x-4)}$$

$$= \frac{x}{4-x} + \frac{2(x-12)}{x(4-x)}$$

LCD: $x(4-x)$

$$= \frac{x \times x + 2(x-12)}{x(4-x)}$$

$$= \frac{x^2 + 2x - 24}{x(4-x)}$$

Simplify further by factorising the numerator:

$$= \frac{(x+6)(x-4)}{x(4-x)}$$

Change the sign: $x-4 = -(4-x)$

$$= \frac{-(x+6)(4-x)}{x(4-x)}$$

$$= \frac{-(x+6)}{x}$$

14. -24 | 20

4 marks

Explanation:

$$(3x^2 - 4y^3)^2 - (x^2 + 2y^3)(x^2 - 2y^3)$$

$$= 9x^4 - 12x^2y^3 - 12x^2y^3 + 16y^6 - (x^4 - 2x^2y^3 + 2x^2y^3 - 4y^6)$$

$$= 9x^4 - 24x^2y^3 + 16y^6 - x^4 + 4y^6 \quad \text{because } -2x^2y^3 + 2x^2y^3 = 0$$

The answer is: $8x^4 - 24x^2y^3 + 20y^6$

The value of $A = -24$ and the value of $B = 20$.



15. false

2 marks

Explanation:

The area of a rectangle:

$$A = l \times b$$

The area is: $x^2 - x - 12$

The length is: $x - 4$

Substitute in the formula:

$$x^2 - x - 12 = (x - 4) \times b$$

Factorise the quadratic trinomial to determine b :

$$x^2 - x - 12 = (x - 4)(x + 3)$$

$$\therefore b = x + 3$$

Circumference of a rectangle:

$$C = 2(l + b)$$

$$C = 2(x - 4 + x + 3)$$

$$C = 2(2x - 1)$$

$$C = 4x - 2$$

16. 5,28 | 5,28 degrees | 5.28

3 marks

Explanation:

$$\cos(3\theta + 10^\circ) = 0,9$$

$$3\theta + 10^\circ = \cos^{-1} 0,9$$

$$3\theta + 10^\circ = 25,84^\circ$$

$$3\theta = 25,84^\circ - 10^\circ$$

$$3\theta = 15,84^\circ$$

$$\therefore \theta = 5,28^\circ$$

On calculator: $2ndF, \cos^{-1}, 0,9, =$

Subtract 10°

Simplify

Divide by 3

17. D: a

4 marks

Explanation:

$$\frac{(ab)^{\frac{2}{3}} \cdot (a^3)^{-\frac{1}{9}} \cdot (b^2)^{-\frac{1}{3}}}{\left(a^{\frac{1}{3}}\right)^{-2}}$$

$$= \frac{a^{\frac{2}{3}} b^{\frac{2}{3}} a^{-\frac{1}{3}} b^{-\frac{2}{3}}}{a^{-\frac{2}{3}}} \quad \text{Exponent Law 4}$$

Simplify the numerator by using Exponent Law 1:

$$\frac{a^{\frac{1}{3}} b^0}{a^{-\frac{2}{3}}}$$

$$= a^{\frac{1}{3} - \left(-\frac{2}{3}\right)} \quad \text{Exponent Law 2}$$

$$= a^{\frac{1}{3} + \frac{2}{3}} \quad \text{Remove brackets and add}$$

$$= a^1 = a$$

18. 64,98;205,02 | 205,02;64,98

3 marks

Explanation:

- Use $\triangle ABC$ to calculate the height of the building:

$$\begin{aligned}\tan \hat{C}_1 &= \frac{AB}{BC} \\ \hat{C}_1 &= \hat{A}_2 \quad \text{alt } \angle \text{s; } AD \parallel BC \\ \hat{C}_1 &= 18^\circ\end{aligned}$$

$$\begin{aligned}\tan 18^\circ &= \frac{AB}{200} \\ AB &= 200 \times \tan 18^\circ \\ AB &= 64,98\end{aligned}$$

The height of the building is $AB = 64,98$.

- Use $\triangle ADE$ to calculate ED and then the height of the tower:

$$\tan \hat{A}_1 = \frac{ED}{AD}$$

$AD = BC = 200$ Distance between tower and building is the same

$$\begin{aligned}\tan 35^\circ &= \frac{ED}{200} \\ ED &= 200 \times \tan 35^\circ \\ ED &= 140,04\end{aligned}$$

The height of the tower:

$$EC = ED + DC$$

$$DC = AB \quad \text{sides of rectangle } ABCD$$

$$EC = 140,04 + 64,98$$

$$EC = 205,02$$

The height of the tower is $EC = 205,02$.

19. true

2 marks

Explanation:

$$(p - 7)(p + 3) = 24$$

Remove brackets:

$$p^2 + 3p - 7p - 21 = 24$$

Simplify and equate to 0:

$$p^2 - 4p - 21 - 24 = 0$$

$$p^2 - 4p - 45 = 0$$

Factorise:

$$(p - 9)(p + 5) = 0$$

Equate each factor to 0:

$$p - 9 = 0 \quad \text{or} \quad p + 5 = 0$$

$$p = 9 \quad \text{or} \quad p = -5$$

20. false

2 marks

Explanation:

Factorise **fully**:

$$\begin{aligned} & (x^2 - 1)^2 - 9 \\ &= [(x^2 - 1) - 3][(x^2 - 1) + 3] && \text{Difference of squares} \\ &= (x^2 - 4)(x^2 + 2) && \text{Simplify each set of brackets} \\ &= (x - 2)(x + 2)(x^2 + 2) && (x^2 - 4) = (x - 2)(x + 2) \end{aligned}$$

The answer is: $(x - 2)(x + 2)(x^2 + 2)$

21. 0;2 | 2;0

3 marks

Explanation:

Solve for x : $2x^2 = 4x$

Add $-4x$:

$$2x^2 - 4x = 4x - 4x$$

$$2x^2 - 4x = 0$$

Factorise:

$$2x(x - 2) = 0$$

Equate each factor to 0:

$$2x = 0 \quad \text{or} \quad x - 2 = 0$$

Solve each equation:

$$x = 0 \quad \text{or} \quad x = 2$$

22. D: $\tan \hat{C} = \frac{BD}{AC}$

4 marks

Explanation:

$\tan \hat{C} = \frac{BD}{AC}$ is incorrect.

In the sketch we have 3 right-angled triangles:

$\triangle ABC$, $\triangle ADB$ and $\triangle DBC$

In $\triangle DBC$:

$$\tan \hat{C} = \frac{BD}{DC}$$

OR

In $\triangle ABC$

$$\tan \hat{C} = \frac{AB}{BC}$$

23. true

2 marks

Explanation:

In $\triangle DBC$:

$$\sin \hat{B}_2 = \frac{\text{opp}}{\text{hyp}} = \frac{DC}{BC}$$

$$\cos \hat{C} = \frac{\text{adj}}{\text{hyp}} = \frac{DC}{BC}$$

$$\therefore \sin \hat{B}_2 = \cos \hat{C}.$$

24. $(4x^2 - 2x + 1)(16x^3 + 1) \mid 2x^2 - 3x - 6$

4 marks

Explanation:

$$\frac{x+2}{4x^2-2x+1} - \frac{8x+1}{16x^3+1}$$
$$= \frac{x+2}{4x^2-2x+1} - \frac{8x+1}{(2x+1)(4x^2-2x+1)}$$

LCD: $(2x+1)(4x^2-2x+1)$

$$= \frac{(x+2)(2x+1) - (8x+1)}{(2x+1)(4x^2-2x+1)}$$

$$= \frac{2x^2 + x + 4x + 2 - 8x - 8}{(2x+1)(4x^2-2x+1)}$$

$$= \frac{2x^2 - 3x - 6}{(2x+1)(4x^2-2x+1)}$$

$$\therefore A = 2x^2 - 3x - 6$$

25. false

2 marks

Explanation:

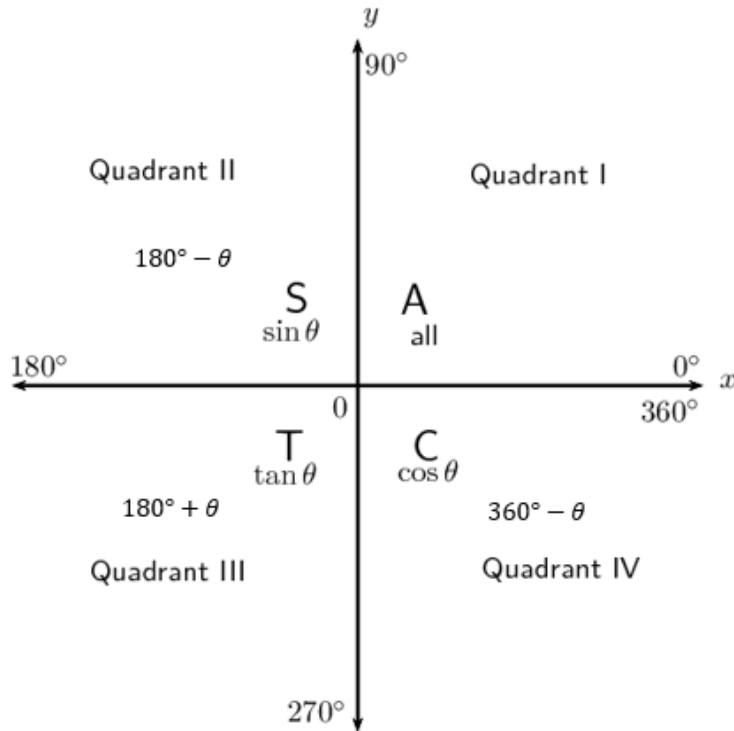
Given: $\sin \theta < 0$ and $\tan \theta > 0$.

This means the quadrant where $\sin \theta$ is **negative** AND $\tan \theta$ is **positive**.

Study the CAST diagram:

$\sin \theta$ is **negative**: Quadrant 3 and 4

$\tan \theta$ is **positive**: Quadrant 1 and 3



We search where they overlap, which is Quadrant 3.

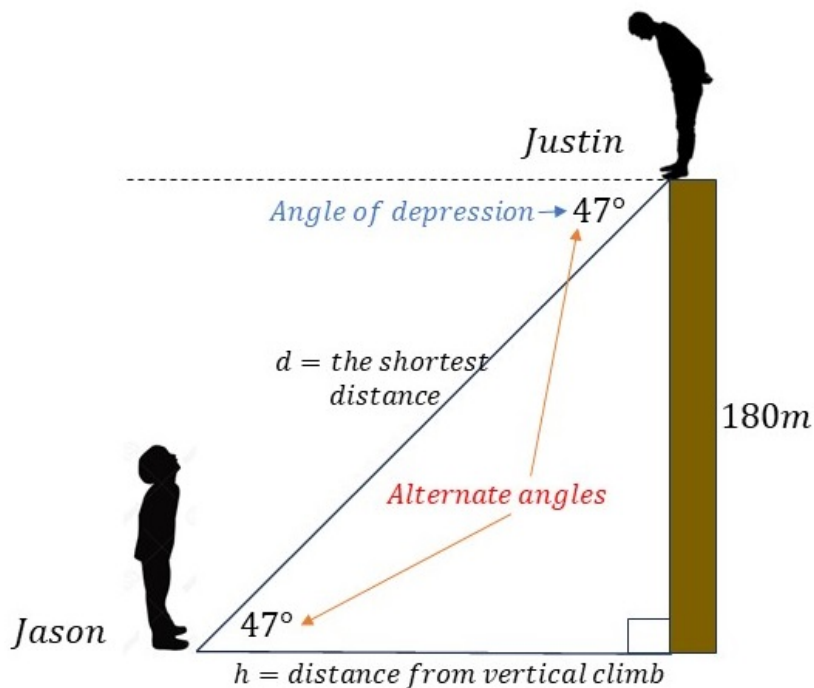
$\therefore$ Quadrant 3: $180^\circ < \theta < 270^\circ$

26. 246,12m | 167,85m

4 marks

Explanation:

Study the sketch:



The angle of depression is equal to the angle where Jason stands. They are **alternate angles**, as indicated on the sketch, because the horizontal lines are parallel. We need an angle **in the triangle** to work with.

- The shortest distance between Jason and Justin is d which is the hypotenuse:

$$\sin 47^\circ = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 47^\circ = \frac{180}{d}$$

$$d \times \sin 47^\circ = 180 \quad \text{Multiply by } d$$

$$d = \frac{180}{\sin 47^\circ} \quad \text{Divide by } \sin 47^\circ$$

$$d = 246,12\text{m}$$

- Jason's distance from the foot of the vertical climb is h :

$$180^2 + h^2 = 246,12^2 \quad \text{Pythagoras}$$

$$h^2 = 246,12^2 - 180^2$$

$$h^2 = 18\,175,0544$$

$$h = 167,85\text{m}$$

Explanation:

- The length of AC :

In $\triangle ABD$:

$$\cos 30^\circ = \frac{AD}{8}$$

$$8 \times \cos 30^\circ = AD$$

$$AD = 4\sqrt{3} = 6,93$$

- The length of BD :

In $\triangle ABD$:

$$\sin 30^\circ = \frac{BD}{8}$$

$$8 \times \sin 30^\circ = BD$$

$$BD = 4$$

- $BD = DC$ Reason: sides opp equal $\angle$ s

Proof:

$$\hat{D}_2 + \hat{B}_2 + \hat{C} = 180^\circ \text{ Int } \angle \text{ s } \triangle$$

$$90^\circ + \hat{B}_2 + 45^\circ = 180^\circ$$

$$\hat{B}_2 = 180^\circ - 90^\circ - 45^\circ$$

$$\therefore \hat{B}_2 = 45^\circ$$

$$\therefore BD = DC \text{ Reason: sides opp equal } \angle \text{ s}$$

- $AC = AD + DC$

$$\text{Therefore } AC = 6,93 + 4,00 = 10,93 \text{ units}$$

28. C: $(2a - 5)(3a + 2)$

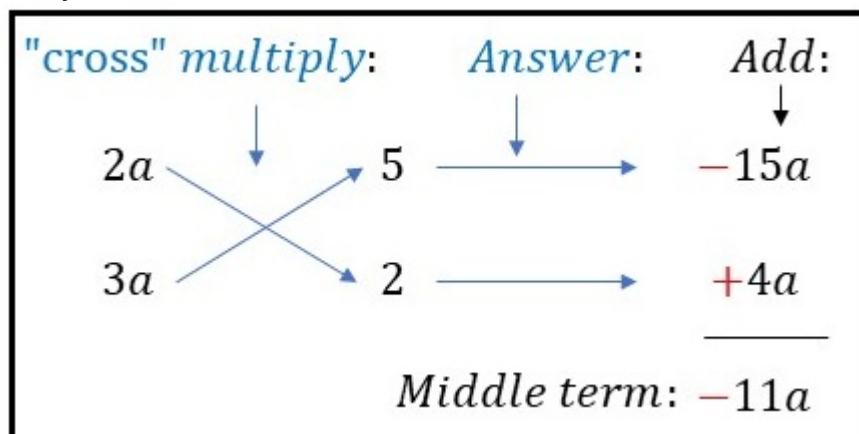
4 marks

Explanation:

Factorise: $6a^2 - 11a - 10$

Consider the factors of $6a^2$ and the factors of 10 and in which way they can be added or subtracted to find the middle term of $-11a$.

Study the sketch:



The answer is: $(2a - 5)(3a + 2)$

29. $(5x-y)(1-y) \mid (1-y)(5x-y) \mid (y-5x)(y-1) \mid (y-1)(y-5x)$

4 marks

Explanation:

$$5x + y^2 - 5xy - y$$

$$= 5x - y - 5xy + y^2 \quad \text{Rearrange the terms}$$

Group the first 2 terms and the last 2 and take out a common factor:

$$= 1(5x - y) - y(5x - y)$$

Take out a common factor of $(5x - y)$:

$$= (5x - y)(1 - y)$$

$$\text{Answer: } (5x - y)(1 - y)$$

The terms can also be arranged like this:

$$5x - 5xy - y + y^2$$

$$= 5x(1 - y) + y(1 - y)$$

$$= (5x - y)(1 - y)$$

The answers will be the same.

Explanation:**Option 1:**

Re-arrange the equation:

$$1 - (x - 1)^2 = 0$$

$$1 = (x - 1)^2 \quad \text{Add } (x - 1)^2 \text{ both sides}$$

$$(x - 1)^2 = 1 \quad \text{Swop the sides}$$

$$\sqrt{(x - 1)^2} = \pm\sqrt{1} \quad \text{Take square root both sides with plus/minus}$$

$$x - 1 = \pm 1$$

$$x - 1 = 1 \quad \text{or} \quad x - 1 = -1$$

$$x = 2 \quad \text{or} \quad x = 0$$

OR**Option 2:**

Multiply the brackets:

$$1 - (x - 1)(x - 1) = 0$$

$$1 - (x^2 - x - x + 1) = 0$$

$$1 - (x^2 - 2x + 1) = 0$$

$$1 - x^2 + 2x - 1 = 0$$

$$-x^2 + 2x = 0$$

$$-x(x - 2) = 0$$

$$-x = 0 \quad \text{or} \quad x - 2 = 0$$

$$x = 0 \quad \text{or} \quad x = 2$$

Total: 105 Marks